Open Channel Flow I - The Manning Equation and Uniform Flow

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COURSE CONTENT

1. Introduction

Flow of a liquid may take place either as open channel flow or pressure flow. Pressure flow takes place in a closed conduit such as a pipe, and pressure is the primary driving force for the flow. For open channel flow, on the other hand the flowing liquid has a free surface at atmospheric pressure and the driving force is gravity. Open channel flow takes place in natural channels like rivers and streams. It also occurs in manmade channels such as those used to transport wastewater and in circular sewers flowing partially full.

In this course several aspects of open channel flow will be presented, discussed and illustrated with examples. The main topic of this course is uniform open channel flow, in which the channel slope, liquid velocity and liquid depth remain constant. First, however, several ways of classifying open channel flow will be presented and discussed briefly.

Open Channel Flow Examples: A River and an Irrigation Canal

2. Topics Covered in this Course

I. Methods of Classifying Open Channel Flow

- A. Steady State of Unsteady State Flow
- B. Laminar or Turbulent Flow
- C. Uniform or Non-uniform Flow
- D. Supercritical, Subcritical or Critical Flow

II. Calculations for Uniform Open Channel Flow

- A. The Manning Equation
- B. The Manning Roughness Coefficient
- C. The Reynold's Number
- D. The Hydraulic Radius
- E. The Manning Equation in S.I. Units
- F. The Manning Equation in Terms of V Instead of Q
- G. The Easy Parameters to Calculate with the Manning Equation
- H. The Hard Parameter to Calculate - Determination of Normal Depth
- I. Circular Pipes Flowing Full or Partially Full
- J. Uniform Flow in Natural Channels

III. Summary

IV. References and Websites

3. Methods of Classifying Open Channel Flow

Open Channel flow may be classified is several ways, including i) steady state or unsteady state, ii) laminar or turbulent, iii) uniform or nonuniform, and iv) subcritical, critical or supercritical flow. Each of these will be discussed briefly in the rest of this section, and then uniform open channel flow will be covered in depth in the rest of the course.

<u>Steady State or Unsteady State Flow:</u> The meanings of the terms steady state and unsteady state are the same for open channel flow as for a variety of other flowing fluid applications. For steady state flow, there are no changes in velocity patterns and magnitude with time at a given channel cross section. Unsteady state flow, on the other hand, does have changing velocity with time at a given cross section. Unsteady state open channel flow takes place when there is a changing flow rate, as for example in a river after a rain storm. Steady state open channel flow takes place when there is a constant flow rate of liquid is passing through the channel. Steady state or nearly steady state conditions are present for many practical open channel flow situations. The equations and calculations in this course will be for steady state flow.

Laminar or Turbulent Flow: Classification of a given flow as either laminar or turbulent is important in several fluid flow applications, such as pipe flow and flow past a flat plate, as well as in open channel flow. In each case a Reynold's number is the criterion used to predict whether a given flow will be laminar or turbulent. Open channel flow is typically laminar for a Reynold's number below 500 and turbulent for a Reynold's number greater than 12,500. A flow with Reynold's number between 500 and 12,500 may be either laminar or turbulent, depending on other conditions, such as the upstream channel conditions and the roughness of the channel walls.

More details on the Reynold's number for open channel flow and its calculation will be given in Section 3, Calculations for Uniform Open Channel Flow. The Reynolds number is greater than 12,500, and thus the flow is turbulent for most practical cases of water transportation in natural or manmade open channels. A notable example of laminar open channel flow is flow of a thin liquid layer on a large flat surface, such as rainfall runoff from a parking lot, highway, or airport runway. This type of flow is often called sheet flow.

Laminar and Turbulent Flow Background: The difference between laminar and turbulent flow in pipes and the quantification of the conditions for each of them, was first observed and reported on by Osborne Reynolds in . His classic experiments utilized injection of dye into a transparent pipe containing a flowing fluid. When the flow was laminar he observed that they dye flowed in a streamline and didn't mix with the rest of the fluid. Under turbulent flow conditions, however, the net velocity of the fluid is in the direction of flow, but there are eddy currents in all directions that cause mixing of the fluid, so that the entire fluid became colored in Reynold's experiments. Laminar and turbulent flow are illustrated for open channel flow in figure 1.

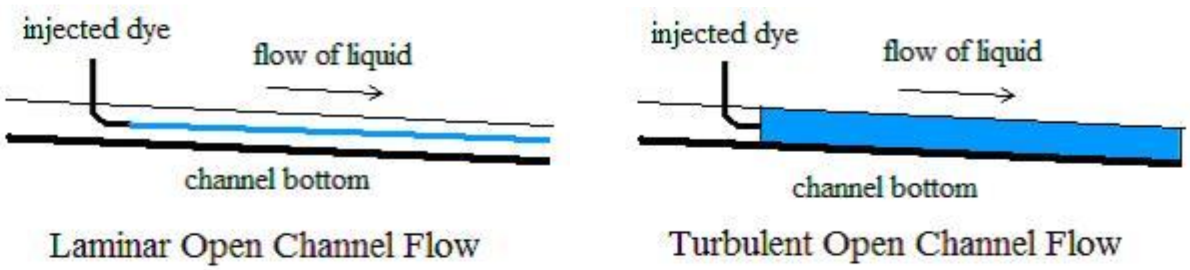


Figure 1. Dye injection into laminar & turbulent open channel flow

Laminar flow is also sometimes called streamline flow. It occurs for flows with high viscosity fluids and/or low velocity and/or high viscosity. Turbulent flow, on the other hand, occurs for fluid flows with low viscosity and/or high velocity.

<u>Uniform or Non-Uniform Flow:</u> Uniform flow will be present in a portion of open channel (called a reach of channel) with a constant flow rate of liquid passing through it, constant bottom slope, and constant cross-section shape & size. With these conditions present, the average velocity of the flowing liquid and the depth of flow will remain constant in that reach of channel. For reaches of channel where the bottom slope, cross-section shape, and/or cross-section size change, non-uniform flow will occur. Whenever the bottom slope and channel cross-section shape and size become constant in a downstream reach of channel, another set of uniform flow conditions will occur there. This is illustrated in Figure 2.

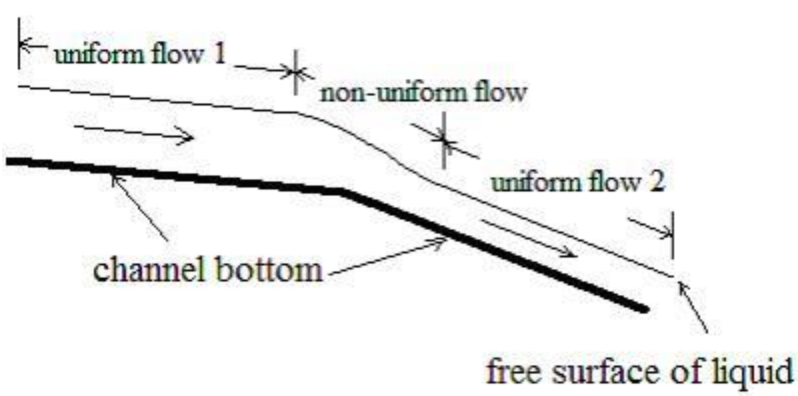


Figure 2. Uniform and Non-uniform Open Channel Flow

Supercritical, Subcitical, or Critical Flow: Any open channel flow will be supercritical, subcritical or critical flow. The differences among these three classifications of open channel flow, however, are not as obvious or intuitive as with the other classifications (steady or unsteady state, laminar or turbulent, and uniform or non-uniform). Your intuition will probably not lead you to expect some of the behaviors for subcritical and supercritical flow and the transitions between them. Subcritical flow occurs with relatively low liquid velocity and relatively deep flow, while supercritical flow occurs with relatively high liquid velocity and relatively shallow flow. The Froude number ($Fr = V/(gl)^{1/2}$) can be used to determine whether a given flow is supercritical, subcritical or critical. Fr is less than one for subcritical flow, greater than one for supercritical flow and equal to one for critical flow. Further discussion of subcritical, supercritical and critical flow is beyond the scope of this course.

4. Calculations for Uniform Open Channel Flow

Uniform open channel flow takes place in a channel reach that has constant channel cross-section size and shape, constant surface roughness, and constant bottom slope. With a constant flow rate of liquid moving though the channel, these conditions lead to flow at a constant liquid velocity and depth, as illustrated in Figure 2.

The Manning Equation is a widely used empirical equation that relates several uniform open channel flow parameters. This equation was developed in 1889 by the Irish engineer, Robert Manning. In addition to being empirical, the Manning Equation is a dimensional equation, so the units must be specified for a given constant in the equation. For commonly used U.S. units the Manning Equation and the units for its parameters are as follows:

$$\mathbf{Q = (1.49/n)A(R_h^{2/3})S^{1/2}} \qquad (1)$$

Where: Q is the volumetric flow rate passing through the channel reach in ft^3/sec.

A is the cross-sectional area of flow perpendicular to the flow direction in ft^2.

S is the bottom slope of the channel* in ft/ft (dimensionless).

n is a dimensionless empirical constant called the Manning Roughness coefficient.

R_h is the hydraulic radius = A/P.

Where: A is the cross-sectional area as defined above in ft^2, and

P is the wetted perimeter of the cross-sectional area of flow in ft.

*Actually, S is the slope of the hydraulic grade line. For uniform flow, however, the depth of flow is constant, so the slope of the hydraulic grade line is the same as that for the liquid surface and the same as the channel bottom slope, so the channel bottom slope is typically used for S in the Manning Equation.

<u>The Manning Roughness Coefficient, n,</u> was noted above to be a dimensionless, empirical constant. Its value is dependent on the nature of the channel and its surfaces. Many handbooks and textbooks have tables with values of n for a variety of channel types and surfaces. A typical table of this type is given as Table 1 below. It gives n values for several man-made open channel surfaces.

What is HYDRAULIC RADIUS ?
Does it need to be round ?

Table 1. Manning Roughness Coefficient, n, for Selected Surfaces

Channel Surface	Manning Roughness Coefficient, n
Asbestos cement	0.011
Brass	0.011
Brick	0.015
Cast-iron, new	0.012
Concrete, steel forms	0.011
Concrete, wooden forms	0.015
Concrete, centrifugally spun	0.013
Copper	0.011
Corrugated metal	0.022
Galvanized Iron	0.016
Lead	0.011
Plastic	0.009
Steel - Coal-tar enamel	0.01
Steel - New unlined	0.011
Steel - Riveted	0.019
Wood stave	0.012

The Reynold's number is defined as $Re = \rho V R_h/\mu$ for open channel flow, where R_h is the hydraulic radius, as defined above, V is the liquid velocity (= Q/A), and ρ and μ are the density and viscosity of the flowing fluid, respectively. Any consistent set of units can be used for R_h, V, ρ, and μ, because the Reynold's number is dimensionless.

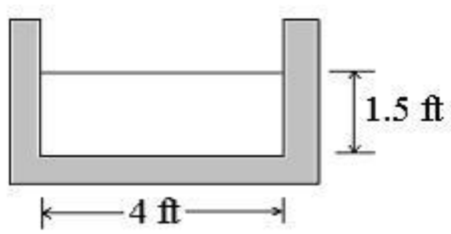
In order to use the Manning equation for uniform open channel flow, the flow must be in the turbulent regime. Forunately, Re is greater than 12,500 for nearly all practical cases of water transport through an open channel, so the flow is turbulent and the Manning equation can be used. Sheet flow, as mentioned above, is a rather unique type of open channel flow, and is the primary example of laminar flow with a free water surface.

The Manning equation doesn't contain any properties of water, however, in order to calculate a value for Reynold's number, values of density and viscosity for the water in question are needed. Many handbooks, textbooks, and websites have tables of density and viscosity values for water as a function of temperature. Table 2 below summarizes values of density and viscosity of water from 32°F to 70°F.

Table 2. Density and Viscosity of Water

Temperature, °F	Density, slugs/ft^3	Dynamic Viscosity, lb-s/ft^2
32	1.940	3.732×10^{-5}
40	1.940	3.228×10^{-5}
50	1.940	2.730×10^{-5}
60	1.938	2.334×10^{-5}
70	1.936	2.037×10^{-5}

Example #1: Water is flowing 1.5 feet deep in a 4 foot wide, open channel of rectangular cross section, as shown in the diagram below. The channel is made of concrete (made with steel forms), with a constant bottom slope of 0.003.
a) Estimate the flow rate of water in the channel. b) Was the assumption of turbulent flow correct ?



Solution: a) Based on the description, this will be uniform flow. Assume that the flow is turbulent in order to be able to use equation (1), the Manning equation. All of the parameters on the right side of equation (1) are known or can be calculated: From Table 1, n = 0.011. The bottom slope is given as: S = 0.003. From the diagram, it can be seen that the cross-sectional area perpendicular to flow is 1.5 ft times 4 ft = 6 ft^2. Also from the figure, it can be seen that the wetted perimeter is 1.5 + 1.5 + 4 ft = 7 ft. The hydraulic radius can now be calculated:

$$R_h = A/P = 6\ ft^2/7\ ft = 0.8571\ ft$$

Substituting values for all of the parameters into Equation 1:

$$Q = (1.49/0.011)(6)(0.8571^{2/3})(0.003^{1/2}) = \underline{\mathbf{40.2\ ft^3/sec = Q}}$$

b) Since no temperature was specified, assume a temperature of 50° F. From Table 2, ρ = 1.94 slugs/ft^3, and $\mu = 2.730 \times 10^{-5}$ lb-s/ft^2. Calculate average velocity, V:

$$V = Q/A = 40.2/6\ ft/sec = 6.7\ ft/sec$$

Reynold's number ($Re = \rho V R_h/\mu$) can now be calculated:

$$Re = \rho V R_h/\mu = (1.94)(6.7)(0.8571)/(2.730 \times 10^{-5}) = 4.08 \times 10^5$$

Since Re > 12,500, this is turbulent flow

The Hydraulic Radius is an important parameter in the Manning Equation. Some common cross-sectional shapes used for open channel flow calculations are **rectangular, circular, semicircular**, **trapezoidal,** and **triangular**. Example #1 has already illustrated calculation of the hydraulic radius for a rectangular open channel. Calculations for the other four shapes will now be considered briefly.

Common examples of gravity flow in a **circular** open channel are the flows in storm sewers, sanitary sewers and circular culverts. Culverts and storm and sanitary sewers usually flow only partially full, however the "worst case" scenario of full flow is often used for hydraulic design calculations. The diagram below shows a representation of a circular channel flowing full and one flowing half full.

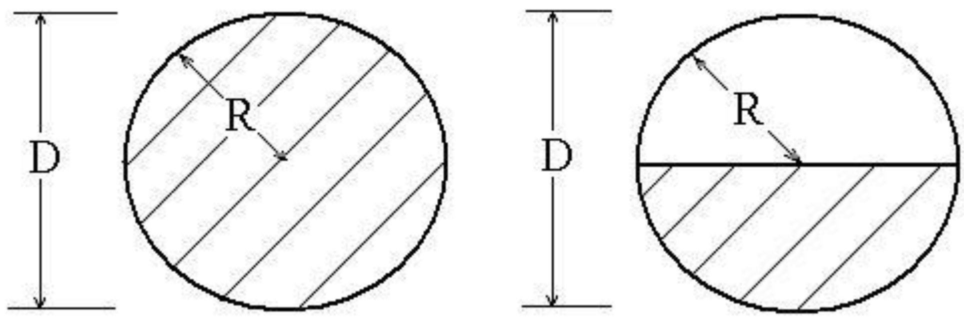


Figure 3. Circular and Semicircular Open Channel Cross-Sections

For a circular conduit with diameter, D, and radius, R, flowing full, the hydraulic radius can be calculated as follows:

The x-sect. area of flow is: $A = \pi R^2 = \pi(D/2)^2 = \pi D^2/4$

The wetted perimeter is: $P = 2\pi R = \pi D$

Hydraulic radius $= R_h = A/P = (\pi D^2/4)/(\pi D)$, simplifying:

For a **circular** conduit flowing full: $\mathbf{R_h = D/4}$ (2)

If a circular conduit is flowing half full, there will be a **semicircular** cross-sectional area of flow, the area and perimeter are each half of the value shown above for a circle, so the ratio remains the same, D/4. Thus:

For a **semicircular** x-section: $\mathbf{R_h = D/4}$ (3)

A **trapezoidal** shape is sometimes used for manmade channels and it is also often used as an approximation of the cross-sectional shape for natural channels. A trapezoidal open channel cross-section is shown in Figure 3 along with the parameters used to specify its size and shape. Those parameters are b, the bottom width; B, the width of the liquid surface; l, the wetted length measured along the sloped side, y; the liquid depth; and α, the angle of the sloped side from the vertical. The side slope is also often specified as: horiz: vert = z:1.

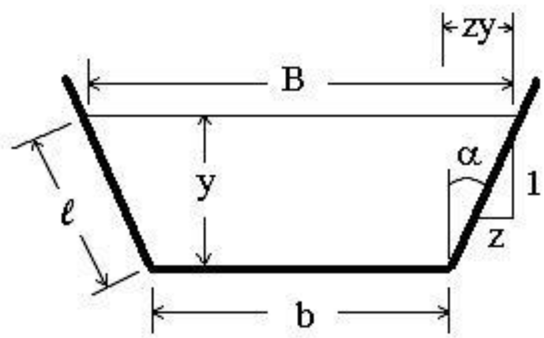


Figure 4. Trapezoidal Open Channel Cross-section

The hydraulic radius for the trapezoidal cross-section is often expressed in terms of liquid depth, bottom width, & side slope (y, b, & z) as follows:

The cross-sectional area of flow = the area of the trapezoid =

$$A = y(b + B)/2 = (y/2)(b + B)$$

From Figure 4, one can see that B is greater than b by the length, zy at each end of the liquid surface. Thus:

$$\mathbf{B = b + 2zy}$$

Substituting into the equation for A:

$$A = (y/2)(b + b + 2zy) = (y/2)(2b + 2zy)$$

Simplifying: $A = by + zy^2$

As seen in Figure 4, the wetted perimeter for the trapezoidal cross-section is:

$$P = b + 2l$$

By Pythagoras' Theorem: $l^2 = y^2 + (yz)^2$ or $l = (y^2 + (yz)^2)^{1/2}$

Substituting into the above equation for P and simplifying:

$$\mathbf{P = b + 2y(1 + z^2)^{1/2}}$$

Thus for a trapezoidal cross-section the hydraulic radius is found by substituting equations (2) & (3) into $R_h = A/P$, yielding the following equation:

For a **trapezoid**: $\mathbf{R_h = (by + zy^2)/(b + 2y(1 + z^2)^{1/2})}$ (4)

A **triangular** open channel cross-section is shown in Figure 5. As would be typical, this cross-section has both sides sloped from vertical at the same angle. Several parameters that are typically used to specify the size and shape of a triangular cross-section are shown in the figure as follows: y, the depth of flow; B, the width of the liquid surface; l, the wetted length measured along the sloped side; and the side slope specified as: horiz : vert = z : 1.

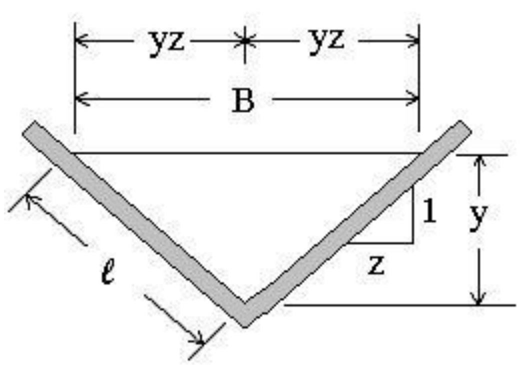


Figure 5. Triangular Open Channel Cross-section

The wetted perimeter and cross-sectional area of flow for a triangular open channel of the configuration shown in Figure 5, can be expressed in terms of the depth of flow, y, and the side slope, z, as follows:

The area of the triangular area of flow is: $A = ½ By$, but from Figure 5:

$B = 2yz$, Thus: $A = ½ (2yz)y$ or simply: $\mathbf{A = y^2z}$

The wetted perimeter is: $P = 2l$ and $l^2 = y^2 + (yz)^2$, solving for l and substituting:

$$\mathbf{P = 2[y^2(1 + z^2)]^{1/2}}$$

Hydraulic radius: $R_h = A/P$

For a **trianglular** x-section: $\mathbf{R_h = y^2z/(2[y^2(1 + z^2)]^{1/2}}$)

Or simplifying: $\mathbf{R_h = yz/[2(1 + z^2)^{1/2}}$] (5)

Example #2: A triangular flume has 10 ft^3/sec of water flowing at a depth of 2 ft above the vertex of the triangle. The side slopes of the flume are: horiz : vert = 1 : 1. The bottom slope of the flume is 0.004. What is the Manning roughness coefficient, n, for this flume?

Solution: From the problem statement: y = 2 ft and z = 1, substituting into Equation (5):

$$R_h = 2(1)/(2[2(1 + 1^2)]^{1/2}) \quad = 0.707 \text{ ft}$$

The cross-sectional area of flow is: $A = y^2z = (2^2)(1) = 4 \text{ ft}^2$

Substituting these values for R_h and A along with given values for Q and S into equation (1) gives:

$$10 = (1.49/n)(4)(0.707^{2/3})(0.004^{1/2})$$

Solving for n: **<u>n = 0.030</u>**

The Manning Equation in SI Units has the constant equal to 1.00 instead of 1.49. The equation and units are as shown below:

$$Q = (1.00/n)A(R_h^{2/3})S^{1/2} \qquad (6)$$

Where: Q is the volumetric flow rate passing through the channel reach in m^3/sec.

A is the cross-sectional area of flow perpendicular to the flow direction in m^2.

S is the bottom slope of the channel in m/m (dimensionless).

n is the dimensionless empirical Manning Roughness coefficient

R_h is the hydraulic radius = A/P.

Where: A is the cross-sectional area as defined above in m^2 and

P is the wetted perimeter of the cross-sectional area of flow in m.

The Manning Equation in terms of V instead of Q: Sometimes it's convenient to have the Manning Equation expressed in terms of average velocity, V, rather than volumetric flow rate, Q, as follows for U.S. units (The constant would be 1.00 for S.I. units.):

$$V = (1.49/n)(R_h^{2/3})S^{1/2} \qquad (7)$$

Where the definition of average velocity, V, is the volumetric flow rate divided by the cross-sectional area of flow:

$$V = Q/A \qquad (8)$$

<u>The Easy Parameters to Calculate with the Manning Equation:</u> Q, V, S, and n are the easy parameters to calculate. If any of these is the unknown, with adequate known information, the Manning equation can be solved for that unknown parameter and then used to calculate the unknown by calculating R_h and substituting known parameters into the equation. This is illustrated for calculation of Q in Example #1 and calculation of n in Example #2. Another example here illustrates bottom slope, S, as the unknown. Then in the next section, we'll take a look at the hard parameter to calculate, normal depth.

Example #3: Determine the bottom slope required for a 12 inch diameter circular storm sewer made of centrifugally spun concrete, if must have an average velocity of 3.0 ft/sec when it's flowing full.

Solution: Solving Equation (6) for S, gives: $S = \{(nV)/[1.49(R_h^{2/3})]\}^2$. The velocity, V, was specified as 3 ft/sec. From Table 1, n = 0.013 for centrifugally spun concrete. For the circular, 12 inch diameter sewer, $R_h = D/4 = ¼$ ft. Substituting into the equation for S gives:

$$S = \{(0.013)(3.0)/[1.49(1/4)^{2/3}]\}^2 = \underline{\mathbf{0.00435 = S}}$$

<u>The Hard Parameter to Calculate - Determination of Normal Depth:</u> For a given flow rate through a channel reach of known shape size & material and known bottom slope, there will be a constant depth of flow, called the **normal depth,** sometimes represented by the symbol, y_o. Determination of the unknown normal depth, y_o, for given values of Q, n, S, and channel size and shape, is more difficult than determination of Q, V, n, or S, as discussed in the previous section. It will be possible to get an equation with y_o as the only unknown, however, in most cases it isn't possible to solve the equation explicitly for y_o, so an iterative or "trial and error" solution is needed. Example #4 illustrates this type of problem and solution.

Example #4: Determine the normal depth for a water flow rate of 15 ft^3/sec, through a rectangular channel with a bottom slope of 0.0003, bottom width of 3 ft, and Manning roughness coefficient of 0.013.

Solution: Substituting specified values into the Manning equation [$Q = (1.49/n)A(R_h^{2/3})S^{1/2}$] gives:

$$15 = (1.49/0.013)(3y_o)((3y_o/(3 + 2y_o))^{2/3})(0.0003^{1/2})$$

Rearranging this equation gives: $3\, y_o(3y_o/(3 + 2y_o))^{2/3} = 7.5559$

There's a unique value of y_o that satisfies this equation, even though the equation can't be solved explicitly for y_o. The solution can be found by an iterative process, that is, by trying different values of y_o until you find the one that makes the left hand side of the equation equal to 7.5559, to the degree of accuracy needed. A spreadsheet such as Excel helps a great deal in carrying out such an iterative solution. The table below shows an iterative solution to Example #4. Trying values of 1, 2, & 3 for y_o, shows that the correct value for y_o lies between 2 and 3. Then the next four trials for y_o, shows that it is between 2.6 and 2.61. The next two entries show that the right hand column is closest to 7.5559 for y_o = 2.60, thus **y_o = 2.60 to three significant figures**.

y_o	$3y_o[3y_o/(3 + 2y_o)]^{2/3}$
1	2.134
2	5.414
3	9.000
2.5	7.184
2.7	7.906
2.6	7.555
2.61	7.580

For a trapezoidal or triangular channel, the procedure for determining normal depth would be the same. In those cases the equations for R_h are a bit more complicated, and the side slope, z, must be specified, but the overall procedure would be like that used in the example above.

<u>Circular Pipes Flowing Full or Parially Full:</u> For a circular pipe, flowing full under gravity flow, such as a storm sewer, $R_h = D/4$, and $A = \pi D^2/4$ can be substituted into the Manning equation to give the following simplified forms:

$$Q = (1.49/n)(\pi D^2/4)((D/4)^{2/3})S^{1/2} \qquad (8)$$

$$V = (1.49/n)((D/4)^{2/3})S^{1/2} \qquad (9)$$

The diameter required for a given velocity or given flow rate at full pipe flow, with known slope and pipe material can be calculated directly, by solving the above equations for D, giving the following two equations:

$$D = 4[Vn/(1.49S^{1/2})]^{3/2} \qquad (10)$$

$$D = \{[4^{5/3}/(1.49\pi)]^{3/8}\}Qn/S^{1/2} = 1.33Qn/S^{1/2} \qquad (11)$$

Calculations for the hydraulic design of storm sewers are typically made on the basis of the circular pipe flowing full under gravity. Storm sewers actually flow less than full much of the time, however, due to storms less intense than the design storm, so there is sometimes interest in finding the flow rate or velocity for a specified depth of flow in a storm sewer of known diameter, slope and n value. Equations are available for these calculations, but they are rather awkward to use, so a convenient to use graph correlating V/V_{full} and Q/Q_{full} to d/D (depth of flow/diameter of pipe), has been prepared and is widely available in handbooks, textbooks and on the internet. That graph is given in Figure 7 below.

The depth of flow, d, and pipe diameter, D, are shown in Figure 6, and Figure 7 gives the correlation between V/V_{full}, Q/Q_{full}, and d/D.

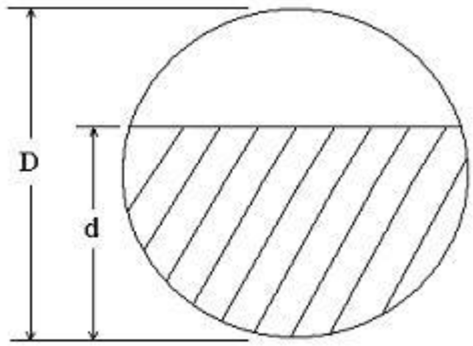


Figure 6. Depth of Flow, d, and Diameter, D, for Partially Full Pipe Flow

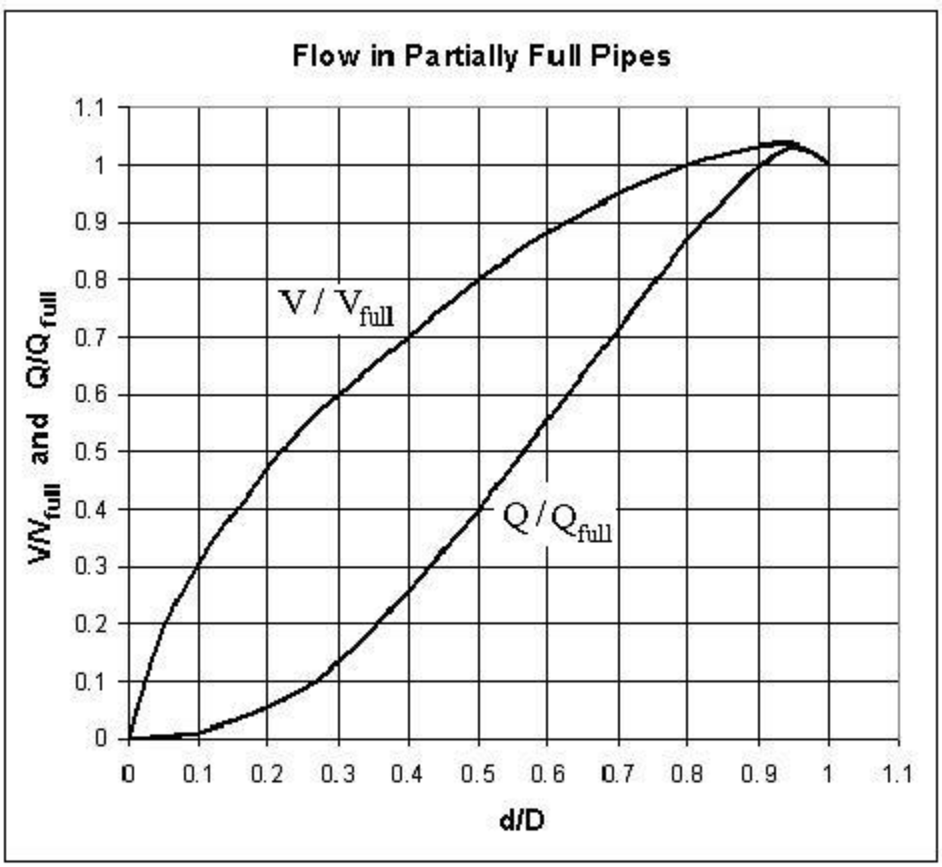


Figure 7. Flow Rate and Velocity Ratios in Pipes Flowing Partially Full

Example #5: Calculate the velocity and flow rate in a 24 inch diameter storm sewer with slope = 0.0018 and n = 0.012, when it is flowing full under gravity.

Solution: From Equation (9):

$V = (1.49/n)((D/4)^{2/3})S^{1/2} = (1.49/0.012)((2/4)^{2/3})(0.0018^{1/2}) =$ **$\underline{3.32 \text{ ft/sec} = V_{full}}$**

Then: $Q = VA = (3.32)(\pi 2^2/4) = 3.32\pi =$ **$\underline{10.43 \text{ cfs} = Q_{full}}$**

Example #6: What would be the velocity and flow rate of water in the storm sewer from Example #5, when it is flowing at a depth of 18 inches?

Solution: d/D = 18/24 = 0.75

From Figure 7: for d/D = 0.75: $Q/Q_{full} = 0.80$ and $V/V_{full} = 0.97$

Thus: $Q = 0.8\ Q_{full} = (0.80)(10.43) =$ **$\underline{8.34 \text{ cfs} = Q}$**

and: $V = 0.97\ V_{full} = (0.97)(3.32) =$ **$\underline{3.22 \text{ ft/sec} = V}$**

Uniform Flow in Natural Channels: The Manning equation is widely applied to flow in natural channels as well as manmade channels. One of the main differences for application to natural channels is less precision in estimating a value for the Manning roughness coefficient, n, due to the great diversity in the type of channels. Another difference is less likelihood of truly constant slope and channel shape and size over an extended reach of channel. One way of handling the problem of determining a value for n is the experimental approach. The depth of flow, channel shape and size, bottom slope and volumetric flow rate are each measured for a channel reach with reasonably constant values for those parameters. Then an empirical value for n is calculated. The value of n can then be used to calculate depth for a given flow or velocity, or to calculate velocity and flow rate for a given depth for that reach of channel.

There are many tables of n values for natural channels in handbooks, textbooks and on the internet. An example is the table on the next two pages from the Indiana Department of Transportation Design Manual, available on the internet at: http://www.in.gov/dot/div/contracts/standards/dm/index.html.

Type of Channel and Description	Minimum	Normal	Maximum
EXCAVATED OR DREDGED			
1. Earth, Straight and Uniform	0.016	0.018	0.020
a. Clean, recently completed	0.018	0.022	0.025
b. Clean, after weathering	0.022	0.025	0.030
c. Gravel, uniform section, clean	0.022	0.027	0.033
2. Earth, Winding and Sluggish			
a. No vegetation	0.023	0.025	0.030
b. Grass, some weeds	0.025	0.030	0.033
c. Dense weeds or aquatic plants in deep channel	0.030	0.035	0.040
d. Earth bottom and rubble sides	0.025	0.030	0.035
e. Stony bottom and weedy sides	0.025	0.035	0.045
f. Cobble bottom and clean sides	0.030	0.040	0.050
3. Dragline, Excavated or Dredged			
a. No vegetation	0.025	0.028	0.033
b. Light brush on banks	0.035	0.050	0.060
4. Rock Cut			
a. Smooth and uniform	0.025	0.035	0.040
b. Jagged and irregular	0.035	0.040	0.050
5. Channel Not Maintained, Weeds and Brush Uncut			
a. Dense weeds, high as flow depth	0.050	0.080	0.120
b. Clean bottom, brush on sides	0.040	0.050	0.080
c. Clean bottom, highest stage of flow	0.045	0.070	0.110
d. Dense brush, high stage	0.080	0.100	0.140
NATURAL STREAM			
1. Minor Stream (top width at flood stage < 100 ft)			
a. Stream on plain			
(1) Clean, straight, full stage, no rifts or deep pools	0.025	0.030	0.033
(2) Same as above, but more stones or weeds	0.030	0.035	0.040
(3) Clean, winding, some pools or shoals	0.033	0.040	0.045
(4) Same as above, but some weeds or stones	0.035	0.045	0.050
(5) Same as above, lower stages, more ineffective slopes and sections	0.040	0.048	0.055
(6) Same as (4), but more stones	0.045	0.050	0.060
(7) Sluggish reaches, weedy, deep pools	0.050	0.070	0.080
(8) Very weedy reaches, deep pools, or floodway with heavy stand of timber and underbrush	0.075	0.100	0.150

NATURAL STREAM (contd.)			
Type of Channel and Description	**Minimum**	**Normal**	**Maximum**
1. Minor Stream (contd.)			
b. Mountain stream, no vegetation in channel, banks usually steep, trees and brush along banks submerged at high stages			
(1) Bottom: gravel, cobbles, and few boulders	0.030	0.040	0.050
(2) Bottom: cobbles with large boulders	0.040	0.050	0.07
2. Floodplain			
a. Pasture, no brush			
(1) Short grass	0.025	0.030	0.035
(2) High grass	0.030	0.035	0.050
b. Cultivated area			
(1) No crop	0.020	0.030	0.040
(2) Mature row crops	0.025	0.035	0.045
(3) Mature field crops	0.030	0.040	0.050
c. Brush			
(1) Scattered brush, heavy weeds	0.035	0.050	0.070
(2) Light brush and trees, in winter	0.035	0.050	0.060
(3) Light brush and trees, in summer	0.040	0.060	0.080
(4)Medium to dense brush, in winter	0.045	0.070	0.110
(5)Medium to dense brush, in summer	0.070	0.100	0.160
d. Trees			
(1) Dense willows, in summer, straight	0.110	0.150	0.200
(2) Cleared land with tree stumps, no sprouts	0.030	0.040	0.050
(3) Same as above, but with heavy growth of sprouts	0.050	0.060	0.080
(4) Heavy stand of timber, a few downed trees, little undergrowth, flood stage below branches	0.080	0.100	0.120
(5) Same as above, but with flood stage reaching branches	0.100	0.120	0.160
3. Major Stream (top width at flood stage > 100 ft). The *n* value is less than that for a minor stream of similar description, because banks offer less effective resistance.			
a. Regular section with no boulders or brush	0.025	n/a	0.060
b. Irregular and rough section	0.035	n/a	0.100

Similar tables are available on many state agency websites. Note that this table gives minimum, normal and maximum values of the Manning Roughness coefficient, n, for a wide range of natural and excavated or dredged channel descriptions.

Example #6: A reach of channel for a stream on a plain is described as clean, straight, full stage, no rifts or deep pools. The bottom slope is reasonably constant at 0.00025 for a reach of this channel. Its cross-section is also reasonably constant for this reach, and can be approximated by a trapezoid with bottom width equal to 7 feet, and side slopes, with horiz : vert equal to 3:1. Using the minimum and maximum values of n in the above table for this type of stream, find the range of volumetric flow rates represented by a 4 ft depth of flow.

Solution to Example #6: From the problem statement, b = 7 ft, S = 0.00025, z = 3, and y = 4 ft. From the above table, item 1. a. (1) under "Natural Stream", the minimum expected value of n is 0.025 and the maximum is 0.033. Substituting values for b, z, and y into equation (4) for a trapezoidal hydraulic radius gives:

$$R_h = [(7)(4) + 3(4^2)]/[7 + (2)(4)(1 + 3^2)^{1/2}] = 2.353 \text{ ft}$$

Also $$A = (7)(4) + 3(4^2) = 76 \text{ ft}^2$$

Substituting values into the Manning Equation $[Q = (1.49/n)A(R_h^{2/3})S^{1/2}]$ gives the following results:

Minimum n (0.025): $Q_{max} = (1.49/0.025)(76)(2.353^{2/3})(0.00025)^{1/2}$

$$\underline{\mathbf{Q_{max} = 126.7 \text{ ft}^3/\text{sec}}}$$

Maximum n (0.033): $Q_{min} = (1.49/0.033)(76)(2.353^{2/3})(0.00025)^{1/2}$

$$\underline{\mathbf{Q_{min} = 95.99 \text{ ft}^3/\text{sec}}}$$

5. Summary

Open channel flow, which has a free liquid surface at atmospheric pressure, occurs in a variety of natural and man-made settings. Open channel flow may be classified as i) laminar or turbulent, ii) steady state or unsteady state, iii) uniform or non-uniform, and iv) critical, subcritical, or supercritical flow. Many practical cases of open channel flow can be treated as turbulent, steady state, uniform flow. Several open channel flow parameters are related through the empirical Manning Equation, for turbulent, uniform open channel flow ($\mathbf{Q = (1.49/n)A(R_h^{2/3})S^{1/2}}$). The use of the Manning equation for uniform open channel flow calculations and for the calculation of parameters in the equation, such as cross-sectional area and hydraulic radius, are illustrated in this course through worked examples.

6. References and Websites

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2. Illinois Department of Transportation Drainage Manual, available on the internet at: http://dot.state.il.us/bridges/brmanuals.html